From Jackiw–Teitelboim Back to Minimal Gravities: Weil-Petersson, Kontsevich, Schwarzschild

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ReNewQuantum Internal Seminar

Based on:

- [PG, Ricardo Schiappa] arXiv:21xx.xxxxx
- [B. Eynard, E. Garcia-Failde, PG, D. Lewański, A. Ooms, Ricardo Schiappa] arXiv:21xx.xxxxx

Outline

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Jackiw-Teitelboim Dilaton Gravity

- We consider 2d dilaton gravity with action

$$S_{\mathrm{JT}} = -\frac{S_0}{4\pi} \underbrace{\int_{\mathcal{M}} \sqrt{g} R}_{\text{topological}} - \frac{1}{2} \underbrace{\int_{\mathcal{M}} \sqrt{g}\phi(R+2)}_{\text{dilaton action}} + (\text{boundary terms})$$

- Dilaton ϕ acts as Lagrange multiplier: $R = -2 \;\rightarrow\; \mathrm{AdS}_2$
- Different topologies weighted by $\left(\mathrm{e}^{S_0}\right)^{2-2g-n} = g_{\mathrm{s}}^{2g+n-2}$
- Holographic dual of SYK model → random ensemble of quantum mechanical models → random matrices

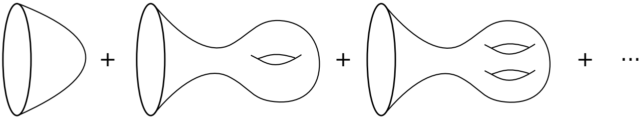
[Sachdev-Ye, Kitaev]
[Saad-Shenker-Stanford]

Euclidean Partition Functions

- Relevant quantities for holography: Euclidean partition functions

$$\langle Z(\beta_1)\cdots Z(\beta_n)\rangle \simeq \sum_{g=0}^{\infty} g_{\rm s}^{2g+n-2} Z_{g,n}(\beta_1)\cdots Z(\beta_n)$$

- Surfaces with n Schwarzian boundaries $+$ g handles



- Asymptotic series $\to$ Resurgence!
- Two-boundary EPF of particular interest: spectral form factor
- Nonperturbative effects are needed!

The Dual Matrix Model

- From disk amplitude: spectral density of dual matrix model:

[Stanford-Witten]
[Saad-Shenker-Stanford]

$$\rho_0(E) = \frac{1}{4\pi^2} \sinh 2\pi\sqrt{E}$$

- From this, Mirzakhani spectral curve: $\frac{\sin 2\pi\sqrt{x}}{4\pi}$
- Weil–Petersson volumes are the building blocks of EPFs:

$$\langle Z(\beta)\rangle \simeq g_s^{-1} Z_{\text{disk}}(\beta) + \sum_{g=1}^{\infty} g_s^{2g-1} \int_0^{\infty} b db\, V_{g,1}(b) Z_{\text{trumpet}}(\beta,\, b)$$

- Euclidean partition functions in the matrix model:

$$\langle Z(\beta_1)\cdots Z(\beta_n)\rangle = \int \mathrm{d}M \mathrm{e}^{N \mathrm{tr} V(M)} \mathrm{tr} \mathrm{e}^{-\beta_1 M} \cdots \mathrm{e}^{-\beta_n M}$$

Correlators and Topological Recursion

- Matrix model correlators:

$$W_n(z_1,\dots,z_n) = 2^n z_1 \cdots z_n \left\langle \mathrm{Tr}\,\frac{1}{z_1^2 - M} \cdots \mathrm{Tr}\,\frac{1}{z_n^2 - M} \right\rangle_{\text{(conn)}}$$

  have a perturbative expansion

$$W_n(z_1,\dots,z_n) \simeq \sum_{g=0}^{+\infty} W_{g,n}(z_1,\dots,z_n)\, g_{\mathsf{s}}^{2g+n-2}$$

- They are computed through topological recursion:

$$W_{g,n}(z_1,J) = \operatorname*{Res}_{z\to 0}\left\{ \frac{\pi}{\sin(2\pi z)} \frac{1}{z_1^2 - z^2} \Big[W_{g-1,h+1}(z,-z,J) + \right.$$
$$\left. + \sum_{\substack{m+m'=g \\ I\sqcup I'=J}}{}' W_{m,|I|+1}(z,I) W_{m',|I'|+1}(-z,I') \Big] \right\}$$

Relation to Weil–Petersson Volumes

- The $W_{g,n}$ are related to Weil–Petersson volumes through a Laplace transform:

$$W_{g,n}(z_1,\ldots,z_n) = \int_0^\infty b_1 db_1 \cdots b_n db_n \, e^{\sum_i z_i b_i} V_{g,n}(b_1,\ldots,b_n)$$

[Eynard-Orantin]

- Topological recursion is equivalent to Mirzakhani's recursion for Weil–Petersson volumes
- Some examples:

$$W_{0,3}(z_1,z_2,z_3) = \frac{1}{z_1^2 z_2^2 z_3^2} \qquad V_{0,3}(b_1,b_2,b_3) = 1$$

$$W_{1,1}(z) = \frac{\pi^2}{12z^2} + \frac{1}{8z^4} \qquad V_{1,1}(b) = \frac{b^2}{48} + \frac{\pi^2}{12}$$

$$W_{0,4}(\vec{z}) = \frac{1}{z_1^2 z_2^2 z_3^2 z_4^2}\left(2\pi^2 + \sum_{i=1}^4 \frac{3}{z_i^2}\right) \qquad V_{0,4}(\vec{b}) = \sum_{i=1}^4 \frac{b_i^2}{2} + 2\pi^2$$

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One-Parameter Transseries for JT Free Energy

- Perturbative series for the free energy of JT gravity:

$$F^{(0)}(g_s) \simeq \sum_{g=0}^{\infty} V_{g,0}\, g_s^{2g-2}$$

- Simple one-parameter transseries ansatz

$$F(g_s, \sigma) = \sum_{n=0}^{\infty} \sigma^n e^{-n\frac{A}{g_s}} F^{(n)}(g_s)$$

- Too naive, but enough for studying one-instanton sector
- Instanton sectors given by

$$F^{(n)}(g_s) \simeq \sum_{g=0}^{\infty} F_g^{(n)} g_s^{\beta_n + g}$$

- Analogous transseries for correlation functions

Resurgent Asymptotics

- Instanton sectors attached to singularities in the Borel plane
- Cauchy's theorem gives us large order relation

$$F_g^{(0)} \simeq \frac{S_1 F_0^{(1)}}{2\pi \mathrm{i}} \frac{\Gamma(2g-\beta)}{A^{2g-\beta}} \left(1 + \frac{A}{2g-\beta-1} \frac{F_1^{(1)}}{F_0^{(1)}} + O(g^{-2})\right) + \\ + \frac{S_1^2 F_0^{(2)}}{2\pi \mathrm{i}} \frac{\Gamma(2g-2\beta)}{(2A)^{2g-2\beta}} \left(1 + \frac{2A}{2g-2\beta-1} \frac{F_1^{(2)}}{F_0^{(2)}} + O(g^{-2})\right) + \cdots$$

- Large g asymptotics entirely encoded in nonperturbative data
  1. Numerical checks of our computations
  2. Large g asymptotics of quantities of interest

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The One-Cut Matrix Model

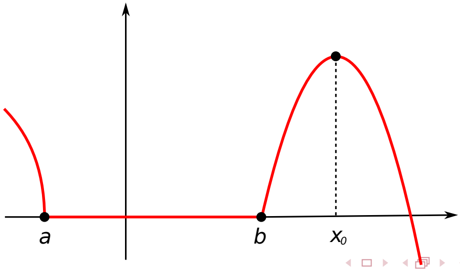
- Consider an $N \times N$ Hermitean one-cut matrix model:

$$Z_N = \frac{1}{\mathrm{vol}\,(\mathrm{U}(N))} \int dM \, e^{-\frac{1}{g_s} \mathrm{Tr}\, V(M)}$$

$$= \frac{1}{N!(2\pi)^N} \int \prod_{i=1}^{N} d\lambda_i \Delta^2(\lambda) e^{-\frac{1}{g_s} \sum_{i=1}^{N} V(\lambda_1)}$$

- At large N, effective potential on an eigenvalue:

$$V_{\text{eff}}(x) = \mathrm{Re} V_{\text{h,eff}}(x), \quad V'_{\text{h,eff}}(x) = y(x) = M(x)\sqrt{(x-a)(x-b)}$$



One-Instanton Contribution to the Free Energy

- One-instanton contribution obtained by placing one eigenvalue on the non-trivial saddle:

[Mariño-Schiappa-Weiss]

$$Z_N^{(1)} = \frac{1}{2\pi} Z_{N-1}^{(0)} \int_{x \in \mathcal{I}} dx \exp\left(-\frac{1}{g_s} V_{\mathrm{h,eff}}(x) + \sum_{n=1}^{\infty} g_s^{n-1} \Phi_n(x) \right)$$

- The $\Phi_n(x)$ are determined by the spectral geometry
- One-instanton contribution to the free energy:

$$F^{(1)} = \frac{Z_N^{(1)}}{Z_N^{(0)}} = \frac{1}{2\pi} \frac{Z_{N-1}^{(0)}}{Z_N^{(0)}} \int_{x \in \mathcal{I}} dx \exp\left(-\frac{1}{g_s} V_{\mathrm{h,eff}}(x) + \sum_{n=1}^{\infty} g_s^{n-1} \Phi_n(x) \right)$$

One-Instanton Contribution to the Free Energy

- We expand the red piece in powers of g_s:

$$\frac{1}{2\pi}\frac{Z_N^{(1)}}{Z_N^{(0)}} = \exp\left(\sum_{n=0}^{\infty} g_s^{n-1}\mathcal{G}_n\right)$$

- We perform the blue integral using the saddle point approximation:

$$\int_{x\in\mathcal{I}} dx(\cdots) = \sqrt{g_s}\, e^{-\frac{V_{\mathrm{h,eff}}(x_0)}{g_s}} \sum_{n=0}^{\infty} g_s^n \mathcal{F}_n$$

- We combine the two pieces to obtain

$$F^{(1)} \simeq \mathrm{i}\sqrt{g_s} S_1 e^{-\frac{A}{g_s}} \sum_{g=0}^{\infty} F_g^{(1)} g_s^g$$

- The instanton action is the expected

$$A = V_{\mathsf{h;eff}}(x_0) - V_{\mathsf{h;eff}}(b) = \int_b^{x_0} dx\, y(x)$$

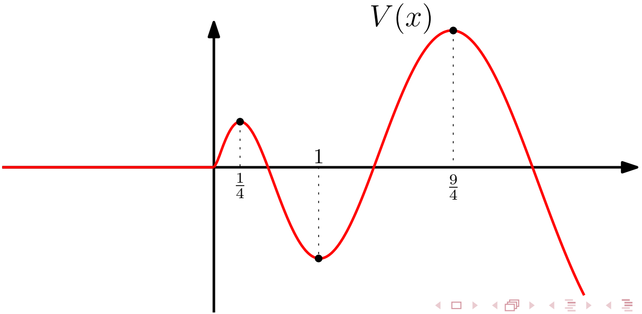
JT Gravity One-Instanton Data

- Also the other terms depend only on the spectral geometry! E.g.:

$$S_1 \cdot F_1^{(1)} = -\mathrm{i}\, \frac{b-a}{4} \sqrt{\frac{1}{2\pi M'(x_0) \left[(x_0 - a)(x_0 - b) \right]^{\frac{5}{2}}}}$$

- For the Mirzakhani spectral curve we obtain

$$V_{\mathrm{h,eff}} = \frac{1}{4\pi^3} \left[\sin(2\pi\sqrt{x}) - 2\pi\sqrt{x} \cos(2\pi\sqrt{x}) \right]$$



JT Gravity One-Instanton Data

- We get an infinite number of non-trivial saddles, with instanton actions:

[Eynard–Garcia-Failde–PG–Lewański–Ooms–Schiappa]

$$A_\ell = (-1)^{\ell+1} \frac{\ell}{4\pi^2}$$

- One-loop and two-loops around the ℓth one instanton:

$$S_1 \cdot F^{(1)}_{\ell,1} = - \frac{\mathrm{i}^{\ell+1}}{\ell^{3/2}\sqrt{2\pi}}$$

$$\frac{F^{(1)}_{\ell,2}}{F^{(1)}_{\ell,1}} = \frac{68(-1+(-1)^\ell) + (-2+3(-1)^\ell)\ell^2\pi^2}{6\ell^3}$$

- Many more with the new approach!

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TR Approach to the One-Instanton Sector

- We introduce the following integrals of the $W_{g,n}$ obtained through topological recursion:

$$F_{g,n}(z) = \overbrace{\int_{-z}^{z} \cdots \int_{-z}^{z}}^{n} W_{g,n}$$

and collect them according to their Euler characteristic:

$$S_\chi(z) = \sum_{2g-1+n=\chi} \frac{F_{g,n}(z)}{n!}, \qquad \chi = 0, 1, \ldots$$

- The key object of our construction is the wave function:

[Eynard]

$$\psi(z, g_s) = \exp\left(\sum_{\chi=0}^{\infty} g_s^{\chi-1} S_\chi(z) \right)$$

- The one-instanton contribution to the free energy is simply given by [Eynard–Garcia-Failde–PG–Lewański–Ooms–Schiappa]

$$F^{(1)} = \frac{1}{2\pi}\int_{\mathcal{I}} \psi(x)\mathrm{d}x = \frac{1}{2\pi}\int_{\mathcal{I}} \exp\left(\frac{1}{g_s}S_0(x) + \sum_{\chi>0} g_s^{\chi-1} S_\chi(x)\right)\mathrm{d}x$$

with the integration done in the saddle point approximation

- same numbers as before, but we easily obtain many more:

$$S_1 \cdot F_0^{(1)} = -\frac{1}{\sqrt{2\pi}}, \qquad \tilde{F}_1^{(1)} = -\frac{68}{3} - \frac{5\pi^2}{6},$$

$$\tilde{F}_2^{(1)} = \frac{12104}{9} + \frac{818\pi^2}{9} + \frac{241\pi^4}{72}$$

$$\tilde{F}_3^{(1)} = -\frac{10171120}{81} - \frac{311672\pi^2}{27} - \frac{175879\pi^4}{270} - \frac{163513\pi^6}{6480} - \frac{29\pi^8}{48}$$

etc.

One-Instanton Sector: Correlators

- The new approach generalizes to correlators easily thanks to the loop insertion operator
- It acts on the $W_{g,n}$:

[Eynard-Orantin]

$$\Delta_z W_{g,n}(z_1,\dots,z_n) = W_{g,n+1}(z_1,\dots,z_n,z)$$

- By extension, on $F_{g,n}$ and S_χ:

$$\Delta_{z_1} F_{g,n}(z) = \Delta F_{g,n}(z,z_1) = \overbrace{\int_{-z}^{z}\cdots\int_{-z}^{z}}^{n} W_{g,n+1}(\cdot,z_1)$$

$$\Delta_{z_1} S_\chi(z) = \Delta S_\chi(z,z_1) = \sum_{2g-1+n=\chi} \frac{\Delta F_{g,n}(z,z_1)}{n!}$$

- It acts as a derivative: $\Delta e^{S(z)} = \Delta S(z)\cdot e^{S(z)}$

One-instanton sector: correlators

- For example, one-instanton contribution to the one-point correlator:

$$W_1^{(1)}(z_1) = \Delta_{z_1} F^{(1)} = \int_{\mathcal{I}} \Delta_{z_1} S(x) e^{S(x)} \mathrm{d}x,$$

- and the two-point correlator:

$$W_2^{(1)}(z_1, z_2) = \int_{\mathcal{I}} \left(\Delta_{z_2} \Delta_{z_1} S(x) + \Delta_{z_1} S(x) \Delta_{z_2} S(x) \right) e^{S(x)} \mathrm{d}x.$$

- After Laplace transform, we obtain one-loop around one-instanton for Weil–Petersson volumes:

[Saad-Shenker-Stanford]
[Eynard–Garcia-Failde–PG–Lewański–Ooms–Schiappa]

$$\frac{V_{1,1}^{(1)}(b)}{V_{1,1}^{(1)}(0)} = \frac{\sinh\left(\frac{b}{2}\right)}{\left(\frac{b}{2}\right)}$$

- We can go to arbitrarily high loops

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Large g Asymptotics of $V_{g,0}$ Weil–Petersson Volumes

- Using resurgent asymptotics we get large g asymptotics of Weil–Petersson volumes:

[Mirzakhani-Zograf]
[Eynard–Garcia-Failde–PG–Lewański–Ooms–Schiappa]

$$V_{g,0}^{(0)} \simeq \frac{\left(4\pi^2\right)^{2g-5/2}}{\sqrt{2}\pi^{3/2}}\Gamma(2g-5/2)\left[1-\frac{1}{4\pi^2(2g-7/2)}\left(\frac{68}{3}+\frac{5\pi^2}{6}\right)+\right.$$
$$+\left(\frac{1}{4\pi^2}\right)^2\frac{1}{(2g-7/9)(2g-9/2)}\left(\frac{12104}{9}+\frac{818\pi^2}{9}+\frac{241\pi^4}{72}\right)+$$
$$\left.+\cdots\right]$$

- We improve on the Mirzakhani-Zograf asymptotics by many orders in g^{-1}

And from Correlators...

- Using resurgent asymptotics we get large g asymptotics of Weil–Petersson $V_{g,1}(b)$ volumes:

[Mirzakhani-Zograf]
[Saad-Shenker-Stanford]
[Eynard–Garcia-Failde–PG–Lewański–Ooms–Schiappa]

$$V_{g,1}^{(0)}(b) \simeq \frac{\left(4\pi^2\right)^{2g-7/2}}{\sqrt{2}\pi^{3/2}}\Gamma(2g-7/2)\frac{\sinh(b/2)}{b/2}\left(1+\cdots\right)$$

- We improve on the Mirzakhani-Zograf asymptotics and on the Saad-Shenker-Stanford result.
- We generalize to any $V_{g,n}(\{b_i\})$:

$$V_{g,n}^{(0)}(\{b_i\}) \simeq \frac{\left(4\pi^2\right)^{2g-n-5/2}}{\sqrt{2}\pi^{3/2}}\Gamma(2g-n-5/2)\prod_{k=1}^{n}\frac{\sinh\left(\frac{b_k}{2}\right)}{b_k/2}\left(1+\cdots\right)$$

Numerical Tests from Resurgent Asymptotics

- We can generate many Weil–Petersson volumes with Zograf's algorithm
- From them, we construct sequences which at $g \to \infty$ converge to the nonperturbative coefficient we want to test:

$$\frac{V_{g+1,0}}{4g^2 V_{g,0}} = \frac{1}{A^2}\left(1 + \frac{1-2\beta}{2g} + O(g^{-2})\right)$$

$$2g\left(A^2 \frac{V_{g+1,0}}{4g^2 V_{g,0}} - 1\right) = 1 - 2\beta + O(g^{-1})$$

$$\frac{A^{2g-\beta} V_{g,0}}{\Gamma(2g-\beta)} = \frac{S_1 F_1^{(1)}}{2\pi i}\left(1 + O(g^{-1})\right)$$

and so on.

Richardson Extrapolation

- We have sequences of the form:

$$S(g) = s_0 + \frac{s_1}{g} + \frac{s_2}{g} + \cdots$$

- The N^{th} Richardson transform of $S(k)$ is defined as

$$\mathrm{RT}_S(g, N) = \sum_{k \geq} \frac{S(g+k)(g+k)^N(-1)^{k+N}}{k!(N-k)!}$$

- This cancels the sub-leading terms in $S(g)$ up to order g^{-N} and accelerates convergence

Numerics for the Instanton Action

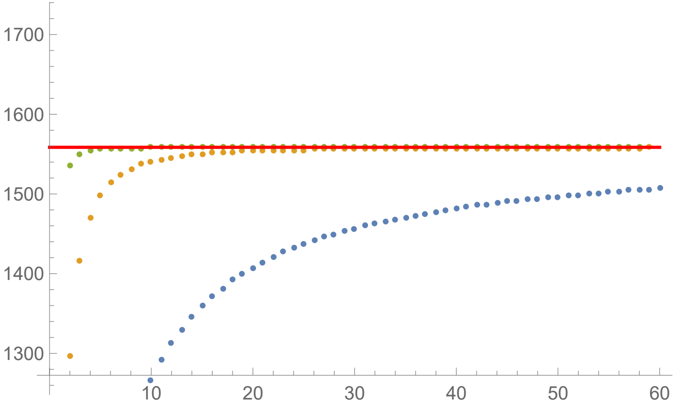

Figure: The sequence $\frac{V_{g+1,\,0}}{4g^2 V_{g,\,0}}$ (blue), its first two Richardson transforms (orange and green),and the predicted value $1/A^2 = 16\pi^2$ (red).

Numerics for the Characteristic Exponent

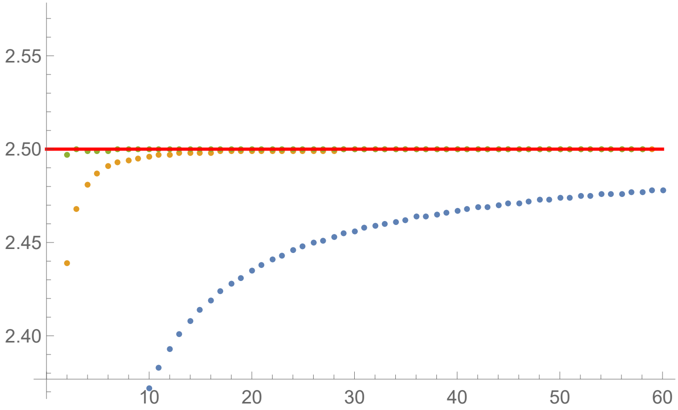

Figure: The sequence $2g\left(A^2 \frac{V_{g+1,\,0}}{4g^2 V_{g,\,0}} - 1\right)$ (blue), its first two Richardson transforms (orange and green), and the predicted value $\beta = 5/2$ (red).

Numerics for the One-Loop Around One-Instanton

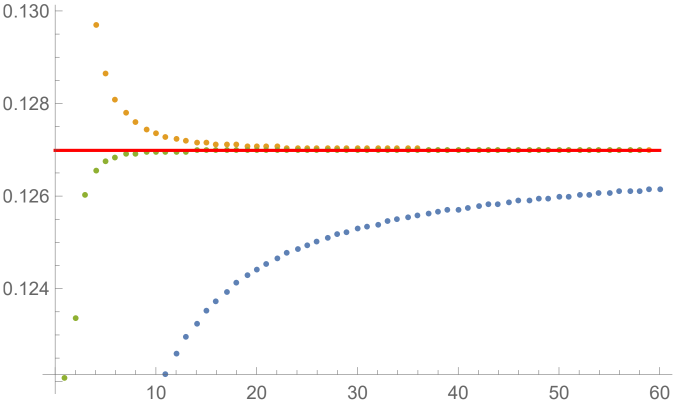

Figure: The sequence $\frac{A^{2g-\beta} F_g^{(0)}}{\Gamma(2g-\beta)}$ (blue), its first two Richardson transforms (orange and green) and the predicted value $\frac{S_1 F_1^{(1)}}{2\pi \mathrm{i}} = \frac{1}{\sqrt{2}\pi^{3/2}}$ (red).

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The Kontsevich Matrix Model

- As a complementary approach, consider the Kontsevich matrix model:

$$Z(\{t_k\}) = \int dM\, e^{-N\,\mathrm{tr}\left[\frac{M^3}{3}+\Lambda M^2\right]} = e^{F(\{t_k\})}$$

depending on KdV times:

$$t_k = \frac{1}{N}\mathrm{tr}\Lambda^{-(2k+1)}, \qquad F(\{t_k\}) \simeq \sum_g g_s^{2g-2} F_g(\{t_k\})$$

- Weil–Petersson volumes can be obtained in the following way:

$$V_{g,n} = \partial_0^n F_g(t_0,\, t_1,\, \ldots)\Big|_{t_0=t_1=0,\, t_k=\frac{(-1)^k}{(k-1)!},\, k\geq 2}$$

- We introduce

$$\mathcal{F}(t_0, t_1) = F(t_0,\, t_1,\, \ldots)\Big|_{t_k=\frac{(-1)^k}{(k-1)!},\, k\geq 2}, \qquad u(t_0, t_1) = \partial_0^2 \psi(t_0, t_1)$$

Transseries Ansatz

- The specific heat satisfies the KdV equation:

$$\partial_1 u = \partial_0 \left(\frac{u^2}{2} + g_s^2 \frac{\partial_0^2 u}{12} \right)$$

- One-parameter transseries for the free energy

$$\mathcal{F}(t_0, t_1, \sigma) = \sum_{n=0}^{\infty} \sigma^n e^{-n \frac{A(t_0,t_1)}{g_s}} \mathcal{F}^{(n)}(t_0, t_1)(g_s)$$

- When plugged into the KdV equation, it gives equation for the instanton action:

[Eynard–Garcia-Failde–PG–Lewański–Ooms–Schiappa]

$$\partial_1 A - u_0^{(0)} \partial_0 A = \frac{1}{12} (\partial_0 A)^3$$

- Supplement it with the Virasoro constraints $L_m Z = 0 \rightarrow$ full Taylor expansion of instanton action

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Minimal Strings as Deformations of JT Gravity

- General 2d dilaton gravity action:

$$S_\phi = -\frac{1}{2}\int drdt\sqrt{g}(R\phi + V(\phi))$$

- The JT gravity potential: $V_{\mathrm{JT}}(\phi) = 2\phi$
- kth Minimal String: Liouville gravity + $(2, 2k-1)$ conformal matter
- It can be recast as dilaton gravity with $V_k(\phi) = \frac{2k-1}{2\pi}\sinh\left(\frac{4\pi\phi}{2k-1}\right)$
- kth Minimal String as deformation of JT gravity:

$$S_\phi = -\frac{1}{2}\int drdt\sqrt{g}\left(R\phi + 2\phi + \frac{4\pi^2\phi^3}{3k^2} + O\left(k^{-3}\right)\right)$$

[Turiaci-Usatyuk-Weng]

Scalar Perturbations - General Potential

- The solution of the e.o.m. for general potential takes the form:

[Louis-Martinez–Kunstatter]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2$$

with

$$f(r) = J(r) + C, \qquad J(x) = \int^x d\rho V(\rho)$$

- We now consider a massless scalar perturbation $\Psi(r,t)$ in the dilaton gravity background. Its Klein-Gordon equation is:

$$\partial_\mu \left(\sqrt{-g}\,\phi\, g^{\mu\nu}\partial_\nu \Psi\right) = 0$$

[Kettner-Kunstatter-Medved]

- Coupling to dilaton inspired by dimensional reduction

Scalar Perturbations - General Potential

- We write an ansatz for the perturbation

$$\Psi(r,t) = \frac{R(r)}{\sqrt{r}} \mathrm{e}^{\mathrm{i}\omega t}$$

- The Klein-Gordon equation turns into a Schrödinger equation for $R(r)$ (Ishibashi-Kodama master equation):

$$\partial_x^2 R(r) + \left[\omega^2 - U(r)\right] R(r) = 0$$

- Derivative is taken w.r.t. the tortoise coordinate

$$x(r) = \int^r d\rho f(\rho)^{-1}$$

- The potential is given by

$$U(r) = \frac{1}{2}\frac{f}{r}\left[f' - \frac{1}{2}\frac{f}{r}\right]$$

- Solving the equation gives the quasinormal spectrum of the background.

The JT Schwarzschild Background

- For JT gravity we have a Schwarzschild-like solution:

$$f(r) = r^2 - r_h^2, \qquad r(x) = -r_h \coth(r_h x)$$

- The potential of the IK master equation is:

$$U_{\mathrm{JT}}(x) = \frac{3r_h^2}{4\sinh^2(r_h x)} + \frac{r_h^2}{4\cosh^2(r_h x)}$$

- The equation can be solved and yields purely imaginary quasinormal modes:

$$\omega_n = -2\mathrm{i} r_h (n+1), \qquad n = 0, 1, 2, \ldots$$

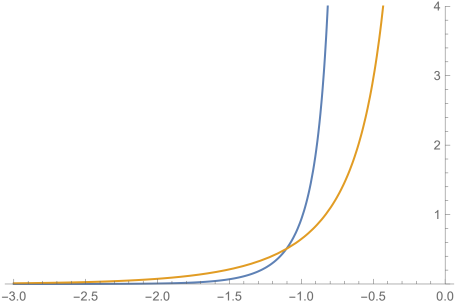
[Bhattacharjee-Sarkar-Bhattacharyya]

The Minimal String Schwarzschild Background

- Also for the kth Minimal String we have a Schwarzschild-like solution: [PG-Schiappa]

$$f(r) = \frac{(2k-1)^2}{8\pi^2}\left[\cosh\left(\frac{4\pi r}{2k-1}\right) - \cosh\left(\frac{4\pi r_h}{2k-1}\right)\right]$$

- The potential of the IK master equation can be obtained analytically.
- See plot of JT potential and $(2,3)$ minimal string:



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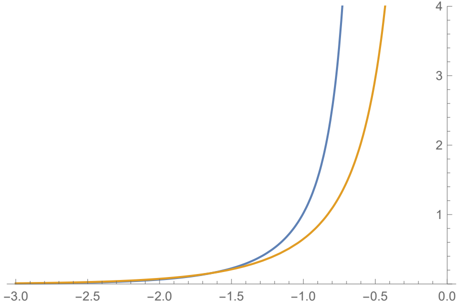

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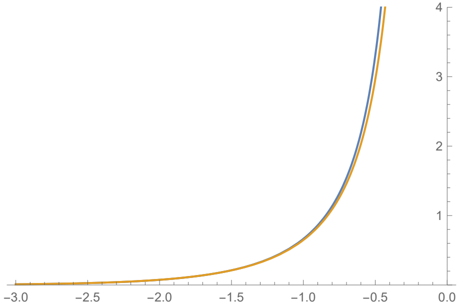
- The potential of the IK master equation can be obtained analytically.
- See plot of JT potential and $(2,51)$ minimal string:



The JT AdS_2 Background

- For JT gravity we have a AdS_2 solution:

$$f(r) = \lambda r^2 + 1, \qquad r(x) = \frac{1}{\sqrt{\lambda}} \tan\left(\sqrt{\lambda}\, x\right)$$

- The potential of the IK master equation is:

$$U_{\text{JT}}(x) = \frac{3\lambda}{4\cos^2(\sqrt{\lambda}\, x)} - \frac{\lambda}{4\sin^2(\sqrt{\lambda}\, x)}$$

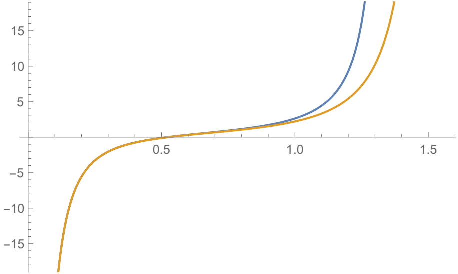
The Minimal String AdS_2 Background

- Also for the kth Minimal String we have a AdS_2-like solution:

[PG-Schiappa]

$$f(r) = (2k-1)^2 \left[\cosh\left(\frac{\sqrt{2\lambda}\, r}{2k-1}\right) - 1\right] + 1$$

- The potential of the Schrödinger eq. can be obtained analytically.
- See plot of JT potential and $(2,3)$ minimal string:



The Minimal String AdS_2 Background

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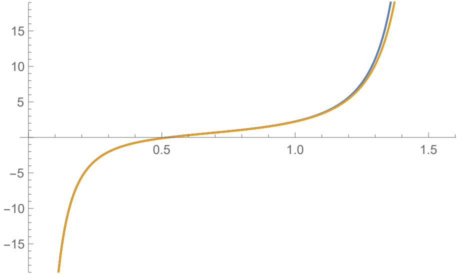

Table of Contents

Summary of the Results

So far we have

- Developed an iterative construction for the computation of one-instanton contributions to the free energy and the correlators of JT gravity
- Used it to obtain large g asymptotics for Weil–Petersson volumes and generalize known results
- Obtained non-trivial information on the instanton action of 2d topological gravity
- Constructed the potentials of IK master equations for minimal string dilaton gravities in different backgrounds

Outlook

There are several ways in which our analysis can be extended:

- Applying our technique to other spectral curves of interest
- considering higher instanton sectors and verifying whether resonance is present
- more on the transseries of general 2d topological gravity
- computation of the quasinormal modes of minimal string backgrounds

Thank you!

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