ERC Synergy grant ***ReNewQuantum***

Phd 2021-2024 in Mathematical Physics

Geometry of integrable systems, topological recursion, quantum curves and asymptotic expansion

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Duration : **3** years, sept 2021 – sept 2024.
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Topic:

This is a very interdisciplinary topic in **mathematical physics**, at the interface between **Mathematics** and **Physics**.
Key words : integrable systems, algebraic and enumerative geometry, combinatorics, topological recursion, resurgence theory, random matrices, string theory, statistical physics, maps.

An integrable system was initially defined as a dynamical system with enough conserved quantities to make it « solvable ». It was rephrased as a set of Poisson-commuting Hamiltonians, and as the existence of a « Tau-function » whose differential is generated by the commuting Hamiltonians. In physics, the Tau function is the partition function. The Tau function is characterized by some relations satisfied by its differential, and in particular a nonlinear equation called « Hirota equation ».
It was observed long ago, that generating functions for several problems in combinatorics or in enumerative geometry, are tau-functions of some integrable systems, for example the Kontsevich integral (generating function for the Kontsevich-Witten intersection numbers in the enumerative geometry of the moduli spaces of Riemann surfaces), is the Korteweg-DeVries (KdV) Tau-function.
More generally, string theory can be rephrased as an enumerative geometry problem : « in how many ways can a Riemann surface of given genus, be holomorphically embedded in a given target space », i.e. measure the volume of a moduli space of pairs of (Riemann surface, holomorphic embedding). Those volumes are extremely hard to compute, and making the link to an integrable system, gives differential equations and other equations, that can provide a way to compute them.

Another link to geometry, is that many examples of Tau-functions happen to be the Theta-function of some algebraic curve, and in some sense, to each algebraic plane curve one can associate a Tau-function. In fact, almost every Tau function, in a certain limit (with a small parameter $\varepsilon \to 0$) behaves as a Theta function asymptotically, and one can study its asymptotic expansion in powers of ε (semi-classical limit, similar to WKB), and find that coefficients of powers of ε, have a geometric interpretation in the geometry of the algebraic plane curve. Vice versa, given an algebraic plane curve, one can construct, from its geometry, a « formal » Tau function as a power series in power of ε.
However a difficulty is that the asymptotic series is usually a divergent series, it doesn't define a function of ε. In other words, we need to understand the resummation procedure. This is related to the « resurgence theory ».

The student will get acquainted to all these concepts. He/she will study several examples of enumerative geometry problems coming either from physics (string theory, statistical physics) or mathematics (algebraic geometry, combinatorics, random matrices). He/she will study the recursion equations that relate the coefficients of the asymptotic series (topological recursion), and work on the question of resummation and resurgence.

Application : send a cv.

Deadline for submission : 30 april 2021. Next selection: may 2021